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Effect of Hole Size on Drainage Time of Bottles

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Abstract—*In this study, water was allowed to drain through holes of different sizes drilled in the cap of an aerated drink's (Appy Fizz) plastic bottle when inverted and exposed to atmospheric pressure. The bottle was clamped on a retort stand to ensure stability. The effect of changing the diameter of the hole on the drainage time was investigated. It was found that the drainage time of a completely filled bottle for different diameters of outlet holes at the bottom could be accurately modeled by considering the bottle to be equivalent to a cylindrical container of uniform area of cross-section having identical height and volume.*

1. INTRODUCTION

Living in a boarding house makes one susceptible to many escapades. A common prank played by students on each other is to make a hole in the base of a closed cold- drink bottle and handing it to a fellow companion. The physics of atmospheric pressure dictates that until the cap is opened (which allows atmospheric pressure to act from above), the water will not flow through the hole. Therefore, as soon as the gullible victim opens the cap, the atmospheric pressure pushes the water through the hole and the trick is successful!

The flow velocity[1] through the hole at the bottom of a cylindrical container which is open to atmosphere at the top, for any instantaneous height of water is given by:

$$v = \sqrt{2gh}$$

where g is the acceleration due to gravity, and h is the instantaneous height of water level. However, this is merely the instantaneous velocity of water escaping from the hole and doesn't feed our curiosity about the relation between drainage time and the diameter of the hole made at the bottom, since even the rate of fall of height of water column varies with time due to the changing area of cross-section of the bottle and the changing velocity of efflux.

We couldn't find any research that modeled the drainage time of an aerated drink bottle of non-cylindrical shape and hence, chose to conduct a research to model this intriguing phenomenon. Hence, we decided to study the effect of hole-size on the drainage time of bottles by modeling a cold-drink bottle of variable area of cross section as a cylinder having identical height and volume but with an average constant area of cross-section given by:

$$A = \frac{V}{H}$$

where A is the average area of cross section of the non-cylindrical shaped bottle, V is the inner volume of the bottle, and H is the height by which the water level can fall inside the bottle till it drains completely.

By applying the equation of continuity and solving the integral, we will get the drainage time of a cylindrical container to be

$$t = \frac{A}{a}\sqrt{\frac{2H}{g}}$$

where a = area of outlet hole in cap when held in inverted position with the cap at the bottom.

Thus, we get the relation between drainage time and diameter of outlet hole as

$$t = \left(\frac{4A}{\pi}\sqrt{\frac{2H}{g}}\right)\frac{1}{d^2}$$

Thus, we expect to find a linear plot between drainage time and $1/d^2$.

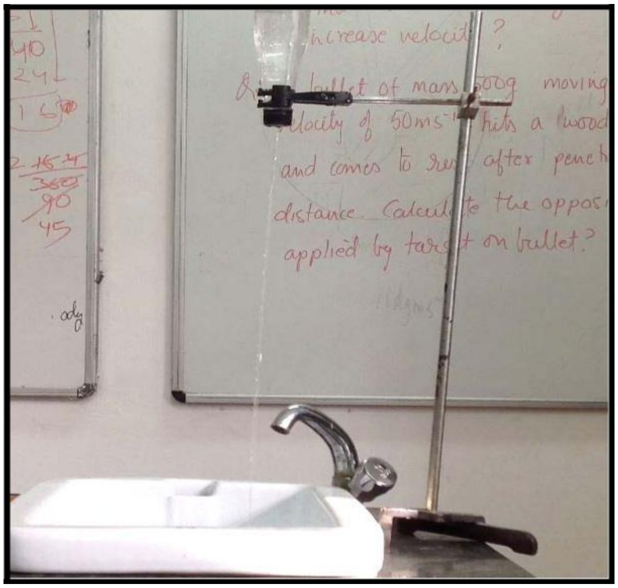
From the slope of this linear plot, we expect to calculate the average area of cross-section of the non-cylindrical shaped bottle and compare its value with what we used in our model.

2. METHOD

We took a 275ml bottle of the popular drink 'Appy Fizz' and made a hole at its base that exposed the water inside to atmospheric pressure. We then took 6 identical caps of the bottles and made holes of different sizes on the caps. The constants of the experiment were:

inner diameter of bottle at neck (minimum diameter) / cm = 2.15 ± 0.01 cm
outer diameter of bottle at base (maximum diameter) / cm = 5.66 ± 0.01cm
height of bottle / cm = 17.7 ± 0.1 cm
inner volume of bottle / cm^3 = 275 ± 1 cm^3

average area of cross section of equivalent cylinder, A / cm^2 = 15.5 ± 0.1 cm^2	
Acceleration due to gravity = 980 cm/s^2	



For the experimental set up, we clamped the bottle of Appy Fizz in inverted position and closed the hole at the base which is now at the top, by pressing tightly with a finger. When we removed our finger from the hole, the atmospheric pressure forced the water to flow through the hole in the cap which is now at the bottom, and we measured the time taken to empty the bottle with a stop watch. We took 5 trials for each of the 6 different diameters of holes.

3. RESULTS AND DISCUSSION

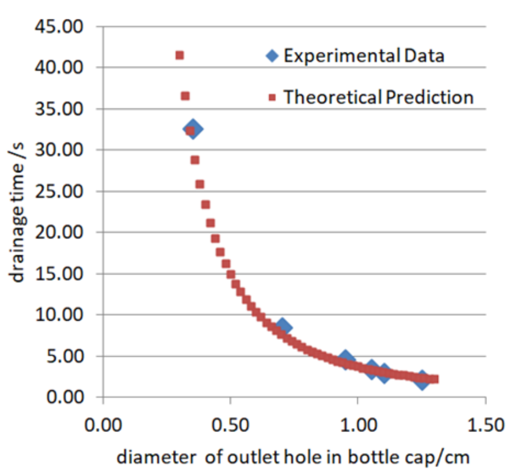


Figure 2 drainage time vs diameter of outlet hole

The experimentally measured values of drainage time were found to lie very close to the values predicted by the model as shown in Figure 2.

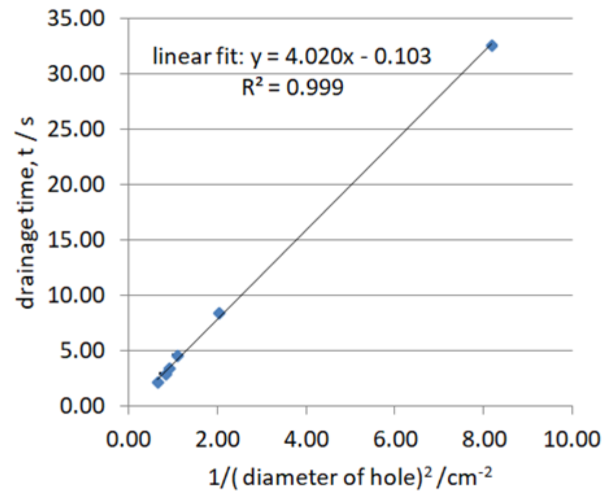


Figure 3 drainage time vs 1/(diameter of outlet hole)^2

As predicted in the hypothesis, the plot of drainage time vs $1/d^2$ gives an excellent linear fit (R^2 = 0.999), and the average area of cross-section of the equivalent cylinder, 'A' as calculated from the slope of the linear fit (16.61 ± 0.02 cm^2) was found to agree very closely with the value predicted by the model (15.5 ± 0.1 cm^2). The fact that the experimental value of A lies in a reasonable range relative to the theoretical value of A shows that our hypothesis is in-fact correct, and the difference caused by the non-uniform curvature of the bottle can be accounted for by the hypothesized model.

4. CONCLUSION

Hence, we can conclude that we can model the effect of outlet hole-size on the time taken for drainage of bottles by treating the non -cylindrically shaped bottle to be equivalent to a cylinder of uniform area of cross section with identical volume and height. Thus, we now have an effective model to study the drainage of a liquid through non-cylindrical bottles of aerated drinks.

REFERENCES

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